Lessons 25 and 26: Stokes' and Divergence Theorems

July 29, 2016

1. Evaluate $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = \langle xy, e^z, \sin(xy) \rangle$ and S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 4$, the paraboloid $z = 10 - x^2 - y^2$, and the plane $z = 1$ with positive orientation.
Answer: 0

2. Evaluate $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = \langle x^3, y^3, z^3 \rangle$ and S is the upper hemisphere $z = \sqrt{1 - x^2 - y^2}$ and the disk $x^2 + y^2 \leq 1$ in the xy-plane.
Answer: $\frac{6\pi}{5}$

3. Evaluate $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = \langle x^3 + y^3, y^3 + z^3, z^3 + x^3 \rangle$ and S is the sphere with center the origin and radius 2.
Answer: $\frac{384\pi}{5}$

1. Verify Stokes' Theorem for the following vector fields and surfaces:

   (a) $\mathbf{F}(x, y, z) = \langle -y, x, -2 \rangle$, S is the cone $z^2 = x^2 + y^2$, $0 \leq z \leq 4$, oriented downward.

   (b) $\mathbf{F}(x, y, z) = \langle -2yz, y, 3x \rangle$, S is the part of the paraboloid $z = 5 - x^2 - y^2$ above the plane $z = 1$, oriented upward

   (c) $\mathbf{F}(x, y, z) = \langle y, z, x \rangle$, S is the hemisphere $x^2 + y^2 + z^2 = 1$, $y \geq 0$, oriented in the direction of the positive y-axis (hint: parameterize S using ϕ and θ)

2. Verify the Divergence Theorem for the following vector fields and regions:

   (a) $\mathbf{F}(x, y, z) = \langle z, y, x \rangle$, E is the solid ball $x^2 + y^2 + z^2 \leq 16$

   (b) $\mathbf{F}(x, y, z) = \langle x^2, -y, z \rangle$, E is the solid cylinder $y^2 + z^2 \leq 9$, $0 \leq x \leq 2$